Conclusions

The variation of the density-viscosity product across the boundary layer has a significant effect on the skin friction and heat transfer. Also, it gives rise to a point of inflexion which can be removed by suction and by increasing the wall temperature. The skin friction and heat transfer are significantly affected by the pressure gradient parameter.

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On Twisting of Orthotropic Plates in a Large Deflection Regime

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Nomenclature

a = plate dimension
h = plate thickness
w = deflection of a point on median surface of plate in direction normal to the undeformed plate
E_x, E_y = Young's moduli of plate in x,y directions
x,y = coordinates in fiber and perpendicular to fiber directions
G_{xy} = shear modulus of plate in x,y plane
μ_{xy}, μ_{yx} = Poisson's ratios, first subscript denotes load direction, second denotes lateral direction
P = applied load
E $= E_x$
F $= E_x/E_y$
μ $= \mu_{xy}$
η $= E_x/G_{xy}$
δ $= (F-\mu^2/\eta F$
ϕ = airy stress function
w_0 = deflection under the load
$\bar{w}_s$ = nondimensional deflection at load in small deflection range
Q_z = transverse shear

Introduction

A PLATE twist test[1] normally is used to determine the shear modulus of an orthotropic plate. One of the requirements for this test to yield reliable results is that the deflection measured should be within a small deflection range. Thick plate specimens are required to insure deflections in the small deflection domain, in view of the other restrictions on the plate dimensions, load applied, and the deflection measured. Thin plate specimens could be used for this test, and, thereby, much material could be saved, provided that the theoretical development existed which included large deflection theory of plates and self weight. To the author's best knowledge, such development does not exist.

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Hence, an attempt is made in this paper to understand the large deflection behavior of orthotropic plates subjected to pure twisting moment.

Analysis

The governing equation in stress function, which is derived by combining the compatibility equation and stress-strain relations for orthotropic plate, is reproduced here from Ref. 2 as follows:

$$F\frac{\partial^4\phi}{\partial x4} + (\eta-2\mu)\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4} = E(w,^2_{xy} - w,_{xx}w,_{yy}) \quad (1)$$

Using strain-stress resultant relations and bending moment-curvature relations in the strain-energy expression for orthotropic plates,[3] the following is obtained:

$$U_{ST} = \tfrac{1}{2}\int_0^a\int_0^a\Big\{\frac{1}{Eh}(N_x^2 - 2\mu N_x N_y + FN_y^2 + \eta N_{xy}^2) + \frac{EFh^3}{12(F-\mu^2)}(w,^2_{xx} + 2\frac{\mu}{F}w,_{xx}w,_{yy} + \frac{1}{F}w,^2_{yy} + 4\delta w,^2_{xy})\Big\}dxdy \quad (2)$$

Knowing the deflection w and induced inplane forces, the strain energy in the plate can be computed using this expression.

The square plate is supported at two opposite corners from below and at a third corner from above. The load is applied at the fourth corner. Thus, the plate is subjected to pure twisting moments distributed at all four edges.

The potential V because of applied load and self weight is

$$V = P(w)_{x=0,y=0} + \int_0^a\int_0^a \rho w dxdy \quad (3)$$

Thus, the total potential energy of the plate is as follows:

$$U = U_{ST} - V \quad (4)$$

The boundary conditions for this plate are

at $x=0$ edge,

$$Q_z(0,y) = 0, \quad 0<y<a \quad (5a)$$

$$M_x(0,y) = 0, \quad 0\le y\le a \quad (5b)$$

$$w(0,a) = 0 \quad (5c)$$

at $x=a$ edge,

$$Q_z(a,y) = 0, \quad 0<y<a \quad (6a)$$

$$M_x(a,y) = 0, \quad 0\le y\le a \quad (6b)$$

$$w(a,0) = w(a,a) = 0 \quad (6c)$$

at $y=0$ edge,

$$Q_z(x,0) = 0, \quad 0<x<a \quad (7a)$$

$$M_y(x,0) = 0, \quad 0\le x\le a \quad (7b)$$

at $y=a$ edge,

$$Q_z(x,a) = 0, \quad 0<x<a \quad (8a)$$

$$M_y(x,a) = 0, \quad 0\le x\le a \quad (8b)$$

The deflection function w satisfying the geometric boundary conditions is assumed as

$$w = A(x-a)(y-a) \quad (9)$$

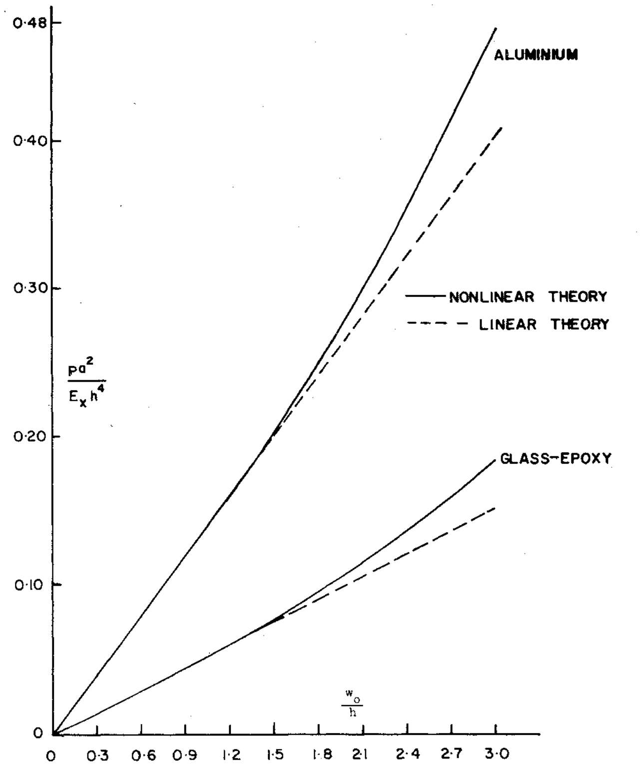

Fig. 1 Load-deflection curves for isotropic (aluminum) and glass-epoxy plates.

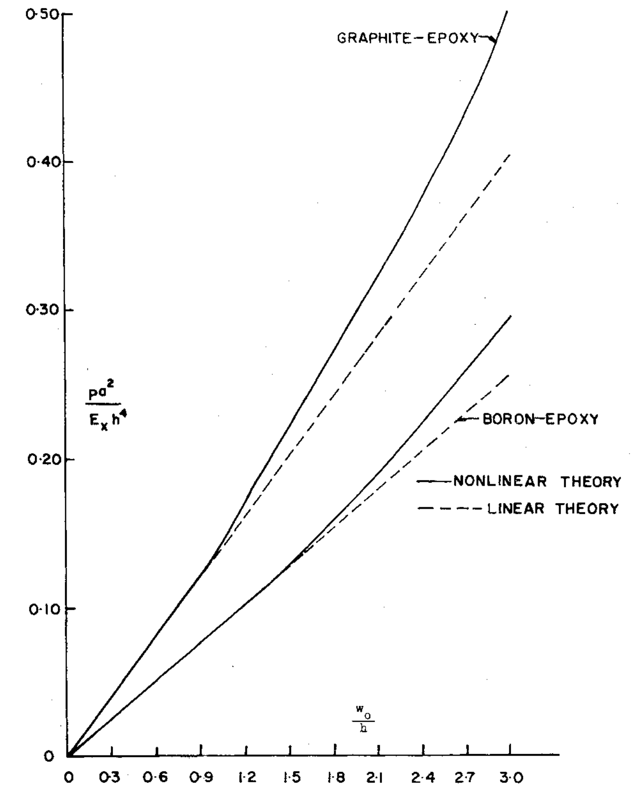

Fig. 2 Load-deflection curves for graphite-epoxy and boron-epoxy plates.

An Airy stress function which satisfies zero normal stresses at every point on the edges and zero average shear stresses on the edges is assumed as

$$\phi = B \sin(\pi x/a) \sin(\pi y/a) \tag{10}$$

Substituting ϕ and w in Eq. (1), and minimizing the error function, the unknown coefficient B is obtained as

$$B = (16/\pi^2 N)\, A^2 \tag{11}$$

where $N = (\pi^4/a^4)(1/E)(1 + F + \eta - 2\mu)$.

Using relations (9-11), the total potential energy of the plate is obtained as

$$U = \frac{32a^2h}{\pi^4 N} A^4 + \frac{G_{xy}}{6} h^3 a^2 A^2 - Pa^2 A - \frac{\rho a^4}{4} A \tag{12}$$

The first term of Eq. (12) represents extensional strain energy because of stretching of the median surface of the plate.

Now applying the principle of total potential energy, the following equation is obtained:

$$\alpha \bar{w}^3 + \beta \bar{w} - \bar{P} - (\bar{\rho}/4) = 0 \tag{13}$$

where

$$\alpha = (128/\pi^8)(1 + F + \eta - 2\mu)^{-1}$$

$$\beta = 1/3\eta, \bar{P} = Pa^2/Eh^4$$

$$\bar{\rho} = \rho a^4/Eh^4, \bar{w} = w_0/h$$

Deflection because of self weight is obtained by solving Eq. (13) without the applied load $\bar{P}$. Also, the deflection in the small deflection range is obtained by deleting the $\bar{w}^3$ term from Eq. (13) and solving it. Hence, $\bar{w}_s = (\bar{P}/\beta) + (\bar{\rho}/4\beta)$. This relation checks with results of Ref. 1 wihtout the self weight.

Conclusion

Load-deflection curves are plotted using Eq. (13), neglecting the contribution of self weight, for isotropic, glass-epoxy, graphite-epoxy, and boron-epoxy plates. Material constants for these plates are taken from Ref. 4 and 5. It is noted from Figs. 1 and 2 that the load-deflection curve is linear up to deflections of the order of thickness of the plates for isotropic, glass-epoxy, graphite-epoxy, and boron-epoxy plates. This means that for the four cases considered, small deflection theory is adequate to predict the deflections, as long as these are of the order of thickness of the plates. Also, it is noted that Eq. (13) can be used to determine the shear modulus of plates by measuring deflections in the large deflection range, provided that the other material constants E_x, E_y, and μ_{xy} are known.

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